HYPOTHESIS TESTING ON HIGH-DIMENSIONAL SPHERES: THE LE CAM APPROACH

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Hypothesis testing in high dimensions has been a most active research topics in the last decade. Both theoretical and practical considerations (associated with concentration of measures and robustness, respectively) make it natural to restrict to *sign tests*, that is, to tests that uses observations only through their directions from a given center. This obviously maps the original Euclidean problem to a spherical one, still in high dimensions. With this motivation in mind, we tackle two testing problems on high-dimensional spheres, both under a symmetry assumption that specifies that the distribution at hand is invariant under rotations with respect to a given axis. More precisely, we consider the problem of testing the null hypothesis of uniformity ("detecting the signal") and the problem of testing the null hypothesis that the symmetry axis coincides with a given direction ("learning the signal direction"). We solve both problems by exploiting Le Cam's asymptotic theory of statistical experiments, in a double- or triple-asymptotic framework. Interestingly, contiguity rates depend in a subtle way on how well the parameters involved are identified as well as on a possible further antipodally-symmetric nature of the distribution. In many cases, strong optimality results are obtained from local asymptotic normality. When this cannot be achieved, it is still possible to establish minimax rate optimality.

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